On a class of degenerate elliptic operators
with unbounded coefficients

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In this talk we consider degenerate elliptic operators of the type

$$\mathcal{A}u(x) = \sum_{i,j=1}^{r} q_{ij}(x) D_{ij} u(x) + \sum_{j=1}^{N} b_{ij} x_j D_i u(x), \qquad x \in \mathbb{R}^N. \tag{1}$$

Under suitable assumptions on the degeneracy of the matrix Q and on its smoothness and on the growth rate at ∞, we show that, for any $f \in C_b(\mathbb{R}^N)$, the Cauchy problem

$$\begin{cases} D_t u(t,x) = \mathcal{A}u(t,x), & t > 0, \quad x \in \mathbb{R}^N, \\ u(0,x) = f(x), & \qquad\qquad x \in \mathbb{R}^N, \end{cases} \tag{2}$$

admits a unique classical solution u. This allows us to associate, in a natural way, a semigroup $\{T(t)\}_{t\geq 0}$ of bounded operators in $C_b(\mathbb{R}^N)$ with problem (2). We provide several uniform estimates for the derivatives of $T(t)f$ up to the third-order, both when f belongs to $C_b^{\theta}(\mathbb{R}^N)$ and when f belongs to some anisotropic space of Hölder continuous functions. As a byproduct we show that the previous estimates can be used to prove Schauder estimates for the distributional solutions to both the nonhomogeneous elliptic equation

$$\lambda u - \mathcal{A}u = f$$

and the nonhomogeneous Cauchy problem

$$\begin{cases} D_t u(t,x) = \mathcal{A}u(t,x) + g(t,x), & t > 0, \quad x \in \mathbb{R}^N, \\ u(0,x) = f(x), & \qquad\qquad x \in \mathbb{R}^N. \end{cases}$$