On a class of degenerate elliptic operators with unbounded coefficients
Luca Lorenzi
Departments di Matematica, Università degli Studi di Parma (Italy)
e-mail: “luca.lorenzi@unipr.it”

In this talk we consider degenerate elliptic operators of the type

\[ Au(x) = \sum_{i,j=1}^{r} q_{ij}(x) D_{ij} u(x) + \sum_{j=1}^{N} b_{ij} x_j D_j u(x), \quad x \in \mathbb{R}^N. \]  

(1)

Under suitable assumptions on the degeneracy of the matrix \( Q \) and on its smoothness and on the growth rate at \( \infty \), we show that, for any \( f \in C^b(\mathbb{R}^N) \), the Cauchy problem

\[ \begin{cases} 
D_t u(t, x) = Au(t, x), & t > 0, \quad x \in \mathbb{R}^N, \\
 u(0, x) = f(x), & x \in \mathbb{R}^N,
\end{cases} \]  

(2)

admits a unique classical solution \( u \). This allows us to associate, in a natural way, a semigroup \( \{T(t)\}_{t \geq 0} \) of bounded operators in \( C^b(\mathbb{R}^N) \) with problem (2). We provide several uniform estimates for the derivatives of \( T(t)f \) up to the third-order, both when \( f \) belongs to \( C^\beta(\mathbb{R}^N) \) and when \( f \) belongs to some anisotropic space of Hölder continuous functions. As a byproduct we show that the previous estimates can be used to prove Schauder estimates for the distributional solutions to both the nonhomogeneous elliptic equation

\[ \lambda u - Au = f \]

and the nonhomogeneous Cauchy problem

\[ \begin{cases} 
D_t u(t, x) = Au(t, x) + g(t, x), & t > 0, \quad x \in \mathbb{R}^N, \\
 u(0, x) = f(x), & x \in \mathbb{R}^N.
\end{cases} \]