The Liouville theorem for conformal maps in the Grushin metric

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Consider in $\mathbb{R}^p \times \mathbb{R}^q$ the Grushin vector fields ∂_{x_j}, $j = 1, \dots, p$ and $(\alpha+1)|x|^\alpha \partial_{y_k}$, $k = 1, \dots, q$, $\alpha > 0$. We prove a Liouville theorem for the classification of conformal maps in the Carnot-Carathéodory distance associated with vector fields of Grushin type. We show that for $p \geq 3$ all conformal maps are obtained as composition of isometries, anisotropic dilations of the form $(x, y) \mapsto (\lambda x, \lambda^{\alpha+1} y)$ and suitable inversions

$$(x, y) \mapsto \left(\frac{x}{\|(x,y)\|^2}, \frac{y}{\|(x,y)\|^{2(\alpha+1)}} \right), \qquad \|(x,y)\| = (|x|^{2(\alpha+1)} + |y|^2)^{1/2(\alpha+1)}$$

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