Sharp two-sided heat kernel estimates for critical Schrödinger operators on bounded domains

LUISA MOSCHINI
Department of Mathematics
University of Rome, "La Sapienza"
moschini@mat.uniroma1.it

Abstract

On a smooth bounded domain $\Omega \subset \mathbb{R}^N$ we consider the Schrödinger operators $-\Delta - V$, with V being the critical borderline potential either $V(x) = (N-2)^2/4\ |x|^{-2}$ or $V(x) = (1/4)\ \mathrm{dist}(x, \partial\Omega)^{-2}$, under Dirichlet boundary conditions. The aim of this talk is to present sharp two-sided estimates on the corresponding heat kernels. To this end we transform the Schrödinger operators into suitable degenerate elliptic operators in divergence form, for which we prove a new parabolic Harnack inequality up to the boundary. In order to succeed in doing this we establish a series of new inequalities such as improved Hardy, logarithmic Hardy Sobolev, Hardy-Moser and weighted Poincaré.

As a byproduct of our technique we are able to answer positively to a conjecture raised by E. B. Davies and to provide on C^2 domains an alternative proof of the local comparison theorem for uniformly parabolic operators obtained by Fabes, Garofalo and Salsa.

The results presented have been obtained in a joint work with S.Filippas and A.Tertikas.