Analytic Semigroups Generated by Elliptic Operators with Boundary Degeneration of First Order

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(Joint work with S. Fornaro, G. Metafune, J. Prüss)

Let Ω be a bounded open subset of $\mathbf{R}^N$ with smooth boundary, ϱ the (smoothed) distance from $\partial\Omega$, and $a = (a_{ij})$ a uniformly elliptic matrix with continuous entries. Define the operator

$$A = -\varrho(x) \sum_{i,j=1}^{N} a_{ij}(x) D_{ij} + \sum_{i=1}^{N} b_i(x) D_i,$$

with b_i continuous, introduce the domains

$$\begin{aligned} D_p(A) &= \left\{ u \in W^{2,p}_{\mathrm{loc}}(\Omega) \cap W^{1,p}_0(\Omega) : \varrho D^2 u \in L^p(\Omega) \right\}, \quad 1 < p < \infty, \\ D_0(A) &= \left\{ u \in C(\overline{\Omega}) \cap \bigcap_{1 \le p < \infty} W^{2,p}_{\mathrm{loc}}(\Omega) \,|\, \sqrt{\varrho}\, \nabla u,\, Au \in C(\overline{\Omega}),\, u_{|\partial\Omega} = 0 \right\} \end{aligned}$$

and set

$$\delta := \min_{\xi \in \partial\Omega} \langle b(\xi), \nu(\xi) \rangle \langle a(\xi)\nu(\xi), \nu(\xi) \rangle^{-1}.$$

We prove that $(-A, D_p(A))$ generates an analytic semigroup in $L^p(\Omega)$, provided that $\delta > -1/p$, and that $(-A, D_0(A))$ generates an analytic semigroup in $C(\overline{\Omega})$, provided that $\delta > -1/2N$.