SEMICONCAVITY OF THE DISTANCE IN THE HEISENBERG GROUP

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Abstract. I will present a joint work with Isabeau Birindelli from the University of Rome. The setting is the Heisenberg Group $\mathbb{H}^n$, that is $\mathbb{C}^n \times \mathbb{R}$ with the group operation:

$$\xi \circ \xi_0 = \Big(z + z_0,\, t + t_0 + 2\,\mathrm{Im}\,(\bar{z}z_0)\Big).$$

The Hörmander vector fields that generate the Heisenberg Algebra are:

$$X_i = \frac{\partial}{\partial x_i} + 2y_i \frac{\partial}{\partial t}, \quad Y_i = \frac{\partial}{\partial y_i} - 2x_i \frac{\partial}{\partial t}$$

which are left-invariant with respect to $\circ$. There are two distances in the Heisenberg Group: the Koranyi distance or smooth distance and the Carnot-Carathéody distance. The Koranyi distance is induced by the norm:

$$\rho = |\xi|_{\mathbb{H}^n} = \Big(|z|^4 + t^2\Big)^{1/4}.$$

While the Carnot-Carathéodory one is defined through the horizontal curves.

Given a close set K we introduce two notions of distance to K. Precisely we shall call the Korany distance:

$$d_K(\xi) = \inf\{|\xi \circ \eta^{-1}|_{\mathbb{H}^n};\, \eta \in K\}.$$

While the other distance is given replacing with the Carnot Carathéodory one. We prove that both these distances are h-semi-concave i.e. they are semiconcave in the geometry of the Heisenberg space if K satisfies the interior sphere condition.